2. REMAINDER THEOREM
FACTOR THEOREM

2.1 The Remainder Theorem

- The difference between equations and identities

$$x^2-4x+4=0 \quad \} \text{ equation}$$

$$\underbrace{(x-2)^2}_{\substack{\text{LHS} \\ \text{(left hand side)}}} \equiv \underbrace{x^2-4x+4}_{\substack{\text{RHS} \\ \text{(right hand side)}}} \quad \} \text{ identity}$$

* note:

$$x^2-4x+4=0$$
$$(x-2)^2=0$$
$$\therefore x=2$$

$\therefore$ only $x=2$ satisfies the equation

* note

for any identity whatever the value you use on the LHS will give you the same value on the RHS

eg. let $x=4$

$$LHS = (4-2)^2 = 4$$
$$RHS = 4^2-4(4)+4 = 4$$

let $x=5$

$$LHS = (5-2)^3 = 9$$
$$RHS = 5^2-4(5)+4 = 9$$

any value of x

$$LHS = RHS$$

* note

$$\left.\begin{aligned} &9x^3-2x^2+5x-2 \\ &6x^4-\frac{3}{4}x+2 \end{aligned}\right\} \text{polynomials}$$

polynomials : sum of terms of the type ax^n, a is a constant, n is a non negative integer

$$\left.\begin{aligned} &x^2-3x^{\frac{1}{2}}+2 \\ &x^3+\frac{2}{x}-4 \end{aligned}\right\} \text{not polynomials!}$$

* note

degree of polynomial
 – the highest power of x

eg. $x^3+2x-4 \to$ degree 3

$2x^2-8 \longrightarrow$ degree 2

* note

For $3x^2-2x+4$

3 : coefficient of x^2
-2 : coefficient of x
4 : coefficient of x^0

i.e. for ax^n
a is called the coefficient of x^n

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* Long Division by example

Find $2x^4 - 3x^3 + x^2 + 1 \div x - 2$

called quotient

$$
\begin{array}{r l}
 & 2x^3 + x^2 + 3x + 6 \\ \cline{2-2}
x-2 & \big)\; 2x^4 - 3x^3 + x^2 + 0x + 1 \\
 & -\;\underline{2x^4 - 4x^3} \\
 & \qquad\quad x^3 + x^2 \\
 & \qquad -\;\underline{x^3 - 2x^2} \\
 & \qquad\qquad\quad 3x^2 + 0x \\
 & \qquad\qquad -\;\underline{3x^2 - 6x} \\
 & \qquad\qquad\qquad\quad 6x + 1 \\
 & \qquad\qquad\qquad -\;\underline{6x - 12} \\
 & \qquad\qquad\qquad\qquad\quad 13
\end{array}
$$

called dividend

called divisor

called remainder

→ stop when degree of remainder (degree of $13 = 0$) is less than the degree of the divisor (degree of $x-2 = 1$)

Double A

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eg. Find the remainder when $4x^3-5x+1$ is divided by $x+3$

$$
\begin{array}{r|l}
 & 4x^2-12x+31 \\
\hline
x+3 & 4x^3+0x^2-5x+1 \\
 & -\underline{4x^3+12x^2} \\
 & \quad -12x^2-5x \\
 & \quad \underline{-12x^2-36x} \\
 & \qquad 31x+1 \\
 & \qquad \underline{31x+93} \\
 & \qquad\quad -92
\end{array}
$$

remainder $= -92$

Now : <u>The Remainder Theorem</u>

> · If the polynomial $f(x)$ is divided by a linear divisor $(x-a)$ the remainder is $f(a)$

Eg. Find the remainder when $4x^3-5x+1$ is divided by $x+3$

Solution : ① Let $f(x)=4x^3-5x+1$

② Let $x+3=0 \therefore x=-3$

③ find $f(-3)$

$$f(-3)=4(-3)^3-5(-3)+1$$
$$=-92 \text{ ※ (see above)}$$

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eg. Find the remainder when $4x^3-5x+1$ is divided by (a) $x-2$
(b) $2x-1$

<u>Solution</u>

(a) let $f(x) = 4x^3 - 5x + 1$

let $x - 2 = 0 \therefore x = 2$

$\therefore f(2) = 4(2)^3 - 5(2) + 1$

$= 32 - 10 + 1$

$= 23$ ⩕

(b) let $2x - 1 = 0$

$\therefore x = 1/2$

$\therefore f(\frac{1}{2}) = 4(\frac{1}{2})^3 - 5(\frac{1}{2}) + 1$

$= -1$ ⩕

eg. The expression $4x^2 - px + 7$ leaves a remainder of -2 when divided by $x-3$. Find p.

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Solution

Let $f(x) = 4x^2 - px + 7$

let $x - 3 = 0 \therefore x = 3$

Given $f(3) = -2$

$\therefore -2 = 4(3)^2 - p(3) + 7$

$-2 = 36 - 3p + 7$

$\therefore 3p = 45$

$p = 15$ //